Financial Frictions - exam solutions (June 9-11, 2021)

1. False. A reduction of entrepreneurs' wealth increases their borrowing needs, and in this model the terms of funding deteriorate with outside borrowing. It is true that some entrepreneurs will be driven out of the market, but the question is about the average quality of *financed* projects and this unambiguously deteriorates.

2. False. It is true that with sufficient outside liquidity the (constrained) optimal liquidation policy can be implemented. But even with no outside liquidity, optimal liquidation can be implemented if entrepreneurs suffer only idiosyncratic liquidity shocks. This requires intermediaries that offer firms insurance by pooling resources and directing them to those that suffer a liquidity shock, see Holmstrom and Tirole (1998).

3. False or Uncertain. Allen and Gale (2000) feature a non-monotone relation between network completeness and systemic risk. A complete network is more resilient than a ring network. But a disconnected network might be more resilient than a ring network. Note that those that responded "true" stating that the complete network is *globally* the most resilient get full points.

4. i) The first best solves:

$$\begin{aligned} \max_{I,c_1^k,c_2^k} \quad & \sum_{k=L,H} P_k \left[\pi_k u(c_1^k) + (1-\pi_k)\rho u(c_2^k)\right], \\ s.t. \quad & \sum_{k=L,H} P_k \pi_k c_1^k = 1 - I, \\ & \sum_{k=L,H} P_k (1-\pi_k) c_2^k = RI. \end{aligned}$$

Since the optimal allocation must completely insure depositors from aggregate liquidity shocks at the level of their individual banks, $c_1^k = c_1^*$, $c_2^k = c_2^*$, and the solution is given by:

$$c_1^* = \frac{1-I}{\pi}, \qquad c_2^* = \frac{IR}{1-\pi}, \qquad k = L, H$$
$$\pi = P_L \pi_L + P_H \pi_H.$$

The first best will then satisfy

$$u'(c_1^*) = \rho R u'(c_2^*),$$

ii) First we note that for $c_1^* \leq c_2^*$, it must be that $\rho R \geq 1$. Note that since you are asked to characterize investment and consumption plans that are state contingent there is an implicit assumption that liquidation is too wasteful to be considered as part of the bank's strategy (credit is given if you solve the fairly more complex problem of characterizing optimal liquidation and reinvestment strategy). State contingent consumptions as a function of initial investment are given by

$$c_1(\pi_i) = \frac{1-I}{\pi_i}, \qquad c_2(\pi_i) = \frac{IR}{1-\pi_i}.$$

In this case the best contract solves

$$\max_I P_L \left[\pi_L u\left(\frac{1-I}{\pi_L}\right) + (1-\pi_L)\rho u\left(\frac{IR}{1-\pi_L}\right)\right] + P_H \left[\pi_H u\left(\frac{1-I}{\pi_H}\right) + (1-\pi_H)\rho u\left(\frac{IR}{1-\pi_H}\right)\right]$$

The first order condition is

$$P_L u'(c_1(\pi_L)) + P_H u'(c_1(\pi_H)) = \rho R \left(P_L u'(c_2(\pi_L)) + P_H u'(c_2(\pi_H))\right),$$

which generically differs from the optimal allocation.

iii) This allocation can be implemented in a decentralized way through an interbank market. Banks of type L have excess liquidity $M_L = 1 - I^* - \pi_L c_1^* = 1 - I^*$, while banks of type H have liquidity needs $M_H = \pi_H c_1^* - (1 - I^*)$. In the aggregate supply and demand match (follow from optimal quantities in a)):

$$p_L M_L = p_H M_H.$$

To find the equilibrium interest rate in the interbank market we look at transactions in period $t = 2$. Banks of type H have excess liquidity that they use to pay their interbank loans. The interbank rate, $1+r$, derives from equating this payment with $(1+r)M_H$, i.e.:

$$(1+r)M_H = RI^* - (1-\pi_H)c_2^*.$$

This gives:

$$1 + r = \frac{RI^* - (1-\pi_H)c_2^*}{\pi_H c_1^* - (1-I^*)} = \frac{(1-\pi)c_2^* - (1-\pi_H)c_2^*}{\pi_H c_1^* - \pi c_1^*} = \frac{c_2^*}{c_1^*}.$$

Note that this is generically different from R.

iv) If banks' liquidity shocks are not observable, then banks of type L would have incentives to pose as H if $1 + r < R$, and banks of type H would have incentives to pose as L if $1 + r > R$ (in both cases to profit from an arbitrage opportunity). Since generically $1 + r \neq R$ the interbank market allocation is not incentive compatible. Thus, the interbank allocation would have to be distorted up to the point that $1 + r = R$, such that no bank has an incentive to lie. To achieve this the contract offers imperfect insurance to depositors (in the sense that $c_1(\pi_H) \neq c_1(\pi_L)$ and $c_2(\pi_H) \neq c_2(\pi_L)$), and is thus second best.

v) A bank run is an equilibrium outcome when a (patient) depositor that expects every other depositor to withdraw in date 1 would be better off withdrawing herself instead of waiting to do so in the final period. For this to be the case the bank would have to be bankrupt if a mass one of depositors withdraws fund early (because otherwise there will be sufficient resources to pay for the lone depositor in the final period). Thus for a run to be an equilibrium the following condition must be satisfied,

$$\pi c_1^* + (1 - \pi c_1^*)L < c_1^*,$$

where the LHS are the resources available to the bank if it liquidates all its long run investment, and the RHS is the deposit demand of a mass one of depositors.

Note that if $c_1^* > 1$ a bank run is always an equilibrium. This will happen when

$$u'(1) > \rho R u'(R).$$

In this model returns are certain. Thus, there is no fundamental reason for the bank to become bankrupt and runs are purely speculative. Thus, in this model runs are inefficient.

vi) We are told that $\pi c_1^* + (1 - \pi c_1^*)L < c_1^*$, so to prevent a run resources have to be raised using taxes on date 1 endowment. A sufficient condition would be to cover the whole difference using taxes, i.e. rasing

$$\tau = c_1^* - (\pi c_1^* + (1 - \pi c_1^*)L < c_1^*),$$

where τ denote the lump sum tax paid by every depositor. So the minimum date 1 endowment e_1 that would prevent a bank run by using a tax-financed deposit insurance is $e_1 = \tau$. If $e_1 < \tau$ for a tax-funded deposit insurance to prevent bank runs it must be that date 1 withdrawals are lower than c_1^* if the mass of depositors seeking their funds at date 1 is larger than π.

5. i) For trader of beliefs π the expected return of a levered investment in the risky asset is

$$R(\pi) = \frac{\pi u + (1-\pi)d - \phi R_f}{p - \phi},$$

where p is the market price of the asset. The marginal trader will be characterized beliefs such that she would be indifferent between purchasing the risky asset or the riskfree one, i.e. her beliefs $\bar{\pi}$ satisfy

$$R(\bar{\pi}) = \frac{\bar{\pi} u + (1-\bar{\pi})d - \phi R_f}{p - \phi} = R_f$$

To find equilibrium price and beliefs $\bar{\pi}$ we use this pricing equation together with the market clearing condition that states

$$\frac{W_0}{p-\phi}(1-\bar{\pi}) = 1.$$

Solving gives

$$\begin{aligned} \bar{\pi} &= \frac{W_0 + \phi - d/R_f}{W_0 + (u-d)/R_f}, \\ p &= \phi + W_0 \frac{u/R_F - \phi}{W_0 + (u-d)/R_f}. \end{aligned}$$

It is clear from the solution that an increase in borrowing limits, ϕ, increases both p and $\bar{\pi}$. Borrowing allows the optimists among traders to increase their leverage and given the fixed supply of the risky asset, market clearing requires a higher price. This higher price drives out of the market the marginal investor that at the current higher price now prefers to save througth the riskfree asset.

ii) The first thing is to note that there is only one marginal trader, not two. This marginal trader will be indifferent between borrowing to purchase the risky asset, short selling the asset, or purchasing the riskfree bond. The key is that the default option is always using the riskless asset, so the return from a short position for the marginal trader is R_f (i.e. $\frac{\bar{\pi}u+(1-\bar{\pi})d}{p} = R_f = \frac{-(\bar{\pi}u+(1-\bar{\pi})d)}{-p}$). Also note that now the maximum a buyer can borrow is given by d/R_f, and the marginal short seller can only lever (in reality it lends in the riskfree market, not borrow) up to u/R_f. This guarantees that in the bad state the payoff of the risky asset allows to cover borrowing for the buyer, and in the good state the payoff of the lending position for short seller covers the return of the short selled asset.

Allowing for short sales will unambiguously reduce the equilibrium asset prices as now pesimists enter the market and the price reflects their beliefs (which previously had no effect on market outcomes). This requires that $\bar{\pi}$ be lower than characterized in (i) above. Note that market clearing should be written as $\frac{W_0}{p-d/R_f}(1-\bar{\pi}) = 1 + \frac{W_0}{p-u/R_f}\bar{\pi} = 1$. With this equation and asset pricing we can find equilibrium prices and the beliefs of the new marginal trader.

iii) The investor's problem is to

$$\max_x \pi \log\left(W_0 R_f + x(u - R_f)\right) + (1-\pi)\log\left(W_0 R_f + x(d - R_f)\right).$$

The first-order condition is therefore

$$\frac{\pi(u - R_f)}{W_0 R_f + x(u - R_f)} = \frac{(1-\pi)(R_f - d)}{W_0 R_f + x(d - R_f)}.$$

Rearranging, this gives the result.

iv) We have

$$\begin{aligned} \text{expected excess return} &= \pi(u - R_f) + (1-\pi)(d - R_f) \\ &= \frac{1}{R_f}(u - R_f)(R_f - d), \end{aligned}$$

where the second equality follows from the above first-order condition which must hold, in equilibrium, with $x = W_0$.

v) The risk-neutral probability is $\pi_u^* = (R_f - d)/(u - d)$. The risk-neutral expected return on the market, $\mathbb{E}^*\widetilde{R}$, is

$$\mathbb{E}^*\widetilde{R} = \pi_u^* u + (1 - \pi_u^*)d = u\frac{R_f - d}{u - d} + d\frac{u - R_f}{u - d} = R_f.$$

The risk-neutral variance is, using $\mathbb{E}^*\widetilde{R} = R_f$,

$$\begin{aligned} var^*\widetilde{R} &= \mathbb{E}^*\widetilde{R}^2 - \left(\mathbb{E}^*\widetilde{R}\right)^2 \\ &= \pi_u^* u^2 + (1-\pi_u^*)d^2 - R_f^2 \\ &= \frac{u^2(R_f - d) + d^2(u - R_f) - R_f^2(u - d)}{u - d} \\ &= (u - R_f)(R_f - d). \end{aligned}$$